

Elementary Sorts

Chapter 2 – Sorting

Algorithms 4th Ed.

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Agenda

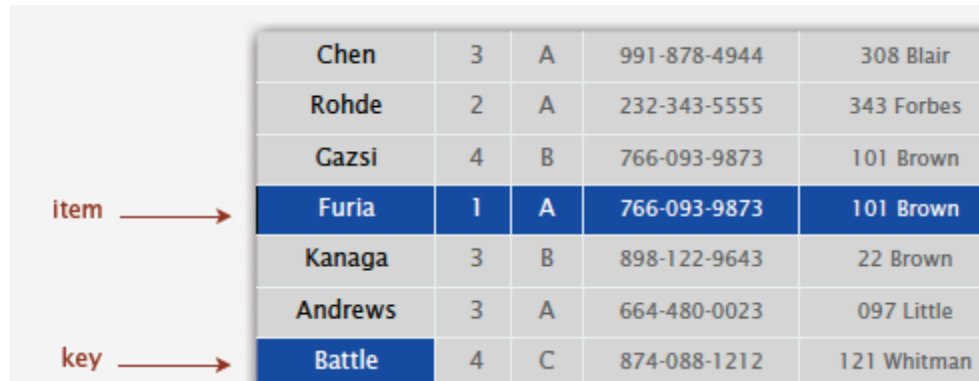
- Comparable
- Selection Sort
- Insertion Sort
- Shell Sort
- Shuffles

Goal of sorting

- The objective of a sorting algorithm is to rearrange the items such that their keys are ordered according to some well-defined ordering rule (usually numerical or alphabetical order)
- Each item contains a key
- Keys as SORTABLE.

Example

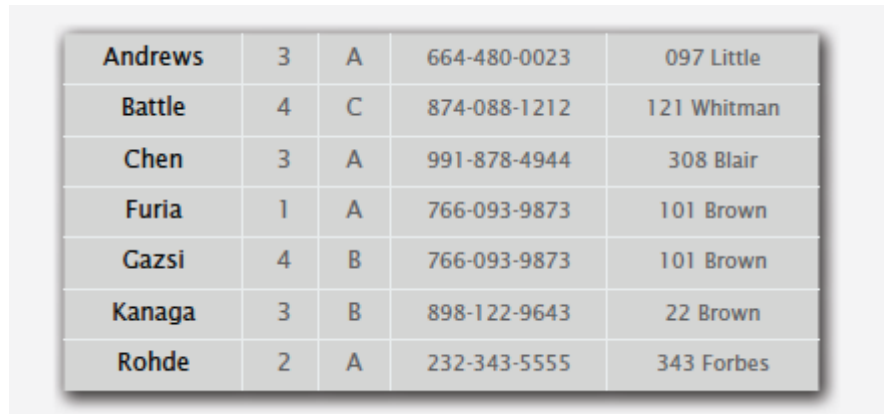
- Unsorted



The diagram shows an unsorted table with 7 rows. The row for 'Furia' is highlighted in blue and labeled 'item' with a red arrow. The row for 'Battle' is also highlighted in blue and labeled 'key' with a red arrow.

Chen	3	A	991-878-4944	308 Blair
Rohde	2	A	232-343-5555	343 Forbes
Gazsi	4	B	766-093-9873	101 Brown
Furia	1	A	766-093-9873	101 Brown
Kanaga	3	B	898-122-9643	22 Brown
Andrews	3	A	664-480-0023	097 Little
Battle	4	C	874-088-1212	121 Whitman

- Sorted



The diagram shows a sorted table with 7 rows. The rows are sorted by the 'key' column (the second column) in ascending order.

Andrews	3	A	664-480-0023	097 Little
Battle	4	C	874-088-1212	121 Whitman
Chen	3	A	991-878-4944	308 Blair
Furia	1	A	766-093-9873	101 Brown
Gazsi	4	B	766-093-9873	101 Brown
Kanaga	3	B	898-122-9643	22 Brown
Rohde	2	A	232-343-5555	343 Forbes

Examples

```
public static void main(String[] args) {
    int n= 10;
    Integer a[] = new Integer[n];

    Random generator = new Random();
    System.out.println("UnSorted Array:");
    for (int i=0;i<n;i++){
        a[i] = generator.nextInt(50);
        System.out.print(a[i] + ", " );
    }

    Insertion.sort(a);
    System.out.println("\nSorted Array:");
    for (int i=0;i<n;i++){
        System.out.print(a[i] + ", ");
    }
}
```

UnSorted Array:

19, 3, 24, 31, 48, 22, 44, 37, 25, 45,

Sorted Array:

3, 19, 22, 24, 25, 31, 37, 44, 45, 48,

```
public static void main(String[] args) {
    int n= 10;
    String a[] = {"apple", "pear",
        "avocado", "banana", "pinapple",
        "peach", "plum", "orange", "blueberry", "kiwi"};

    System.out.println("UnSorted Array:");
    for (int i=0;i<n;i++){
        System.out.print(a[i] + ", " );
    }

    Insertion.sort(a);
    System.out.println("\nSorted Array:");
    for (int i=0;i<n;i++){
        System.out.print(a[i] + ", ");
    }
}
```

UnSorted Array:

apple, pear, avocado, banana, pinapple, peach, plum, orange, blueberry, kiwi,

Sorted Array:

apple, avocado, banana, blueberry, kiwi, orange, peach, pear, pinapple, plum,

<http://algs4.cs.princeton.edu/21elementary/Insertion.java>

Examples

```
public static void main(String[] args) {  
    int n= 10;  
    File directory = new File(".");  
    File[] files = directory.listFiles();  
    Insertion.sort(files);  
    System.out.println("Sorted Array:");  
    for (int i=0;i<n;i++){  
        System.out.print(files[i].getName() + ", " );  
    }  
}
```

Sorted Array:

.classpath, .project, .settings, 16Kints.txt, 1Kints.txt, 1Mints.txt, 2Kints.txt, 32Kints.txt, 4Kints.txt, 8Kints.txt

Comparable Interface

- Q.How does the same sort() method in previous examples work with Files, Strings, Doubles???
- A.They all implement the Comparable interface. (Remember interfaces from 1st Week)
- Sometimes known as “Callback”

Comparable Example

client

```
public static void main(String[] args) {
    int n= 10;
    File directory = new File(".");
    File[] files = directory.listFiles();
    Insertion.sort(files);
    System.out.println("Sorted Array:");
    for (int i=0;i<n;i++){
        System.out.print(files[i].getName() + ", " );
    }
}
```


object implementation

```
public class File
implements Comparable<File>
{
    ...
    public int compareTo(File b)
    {
        ...
        return -1;
        ...
        return +1;
        ...
        return 0;
    }
}
```

Comparable interface (built in to Java)

```
public interface Comparable<Item>
{
    public int compareTo(Item that);
}
```

key point: no dependence
on File data type



sort implementation

```
public static void sort(Comparable[] a)
{
    int N = a.length;
    for (int i = 0; i < N; i++)
        for (int j = i; j > 0; j--)
            if (a[j].compareTo(a[j-1]) < 0)
                exch(a, j, j-1);
            else break;
}
```


Total Order

- `compareTo()` must implement a *total order*. It must be:
 - *Reflexive* (for all v , $v = v$)
 - *Antisymmetric* (for all v and w , if $v < w$ then $w > v$ and if $v = w$ then $w = v$)
 - *Transitive* (for all v , w , and x , if $v \leq w$ and $w \leq x$ then $v \leq x$)
- Example of something that doesn't have Total Order
 - Rock, Paper, Scissors game...

Implementing Comparable Interface

- Must implement `compareTo()` such that `v.compareTo(w)`
 - Returns a negative integer if `v` is less than `w`
 - Returns a positive integer if `v` is greater than `w`
 - Returns 0 if `v` is equal to `w`
 - Is a Total Order



less than (return -1)



equal to (return 0)



greater than (return +1)

Date Class Example

```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

    public Date(int m, int d, int y)
    {
        month = m;
        day   = d;
        year  = y;
    }

    public int compareTo(Date that)
    {
        if (this.year < that.year ) return -1;
        if (this.year > that.year ) return +1;
        if (this.month < that.month) return -1;
        if (this.month > that.month) return +1;
        if (this.day   < that.day   ) return -1;
        if (this.day   > that.day   ) return +1;
        return 0;
    }
}
```



only compare dates
to other dates

<http://algs4.cs.princeton.edu/21elementary/Date.java>

Selection Sort

- General Idea:
 - Iterate through an array of n items $a[n-1]$, starting at $i=0$
 - In iteration i , find the index of the smallest remaining entry
 - Swap $a[i]$ and $a[\text{min}]$
- http://en.wikipedia.org/wiki/Selection_sort
- <http://algs4.cs.princeton.edu/lectures/21DemoSelectionSort.mov>

Selection Sort – Java Implementation

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int min = i;
            for (int j = i+1; j < N; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { return v.compareTo(w) < 0; }

    private static void exch(Comparable[] a, int i, int j)
    {
        Comparable swap = a[i];
        a[i] = a[j];
        a[j] = swap;
    }
}
```

Selection Sort Efficiency

- **Input size metric:** length of Array, N .
- **Is running cost the same for different inputs of same size?** YES
- **Basic Operation:** Comparison (the “if” statement)
- $(N-1) + (N-2) + (N-3) + \dots + 2 + 1$ compares and at most N exchanges.
- $\sim n^2$ running time (classified as quadratic)

Insertion Sort

- General Idea:
 - Iterate through an array of n items $a[n-1]$, starting at $i=0$
 - In iteration i , swap $a[i]$ with each larger entry to its left
 - http://en.wikipedia.org/wiki/Insertion_sort
 - <http://algs4.cs.princeton.edu/lectures/21DemolnsertionSort.mov>

Insertion Sort

		a[]										
i	j	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
1	0	O	S	R	T	E	X	A	M	P	L	E
2	1	O	R	S	T	E	X	A	M	P	L	E
3	3	O	R	S	T	E	X	A	M	P	L	E
4	0	E	O	R	S	T	X	A	M	P	L	E
5	5	E	O	R	S	T	X	A	M	P	L	E
6	0	A	E	O	R	S	T	X	M	P	L	E
7	2	A	E	M	O	R	S	T	X	P	L	E
8	4	A	E	M	O	P	R	S	T	X	L	E
9	2	A	E	L	M	O	P	R	S	T	X	E
10	2	A	E	E	L	M	O	P	R	S	T	X

entries in gray
do not move

entry in red
is a[j]

entries in black
moved one position
right for insertion

Trace of insertion sort (array contents just after each insertion)

Insertion Sort Implementation

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0; j--)
                if (less(a[j], a[j-1]))
                    exch(a, j, j-1);
                else break;
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

    private static void exch(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

Insertion Sort Efficiency

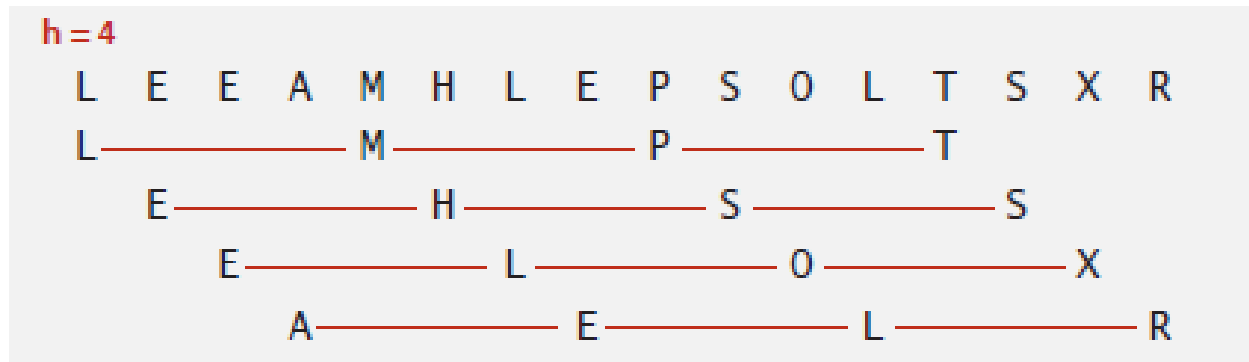
- Input Size Metric : Length of array, N .
- Is running cost the same for different inputs of same size? NO
- Best Case: Array already sorted so $N-1$ compares and 0 exchanges (linear)
- Worst Case: Array in descending order(wrong way sorted) $\frac{1}{2} N^2$ compares and $\frac{1}{2} N^2$ exchanges (quadratic)
- Average Case: For a random ordered class
approx $\frac{1}{4} N^2$ compares and $\frac{1}{4} N^2$ exchanges on average.

Insertion Sort and Partially Sorted Arrays

- Efficiency class of selection and insertion are the same for random arrays
- However, insertion sort works better for “partially sorted arrays” where:
 - each entry is not far from final position
 - Small array appended to larger sorted array
 - Array with only a few elements out of place

Shell Sort

- General Idea:
 - Same as insertion sort but move entries more than one position at a time.
- Known as h-sorting. Swap $a[i]$ with larger entry h positions to the left



Example

- Insertion sort with step size h (red entry indicates $a[i]$, black entries have moved h to left, grey entries stay the same)

3-sorting an array

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

Shell Sort – 7,3,1

(red entry indicates $a[i]$, black entries have moved h to left, grey entries stay the same)

input

S O R T E X A M P L E

7-sort

S O R T E X A M P L E
M O R T E X A S P L E
M O R T E X A S P L E
M O L T E X A S P R E
M O L E E X A S P R T

3-sort

M O L E E X A S P R T
E O L M E X A S P R T
E E L M O X A S P R T
E E L M O X A S P R T
A E L E O X M S P R T
A E L E O X M S P R T
A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T

1-sort

A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T
A E E L O P M S X R T
A E E L O P M S X R T
A E E L M O P S X R T
A E E L M O P S X R T
A E E L M O P S X R T
A E E L M O P R S T X

result

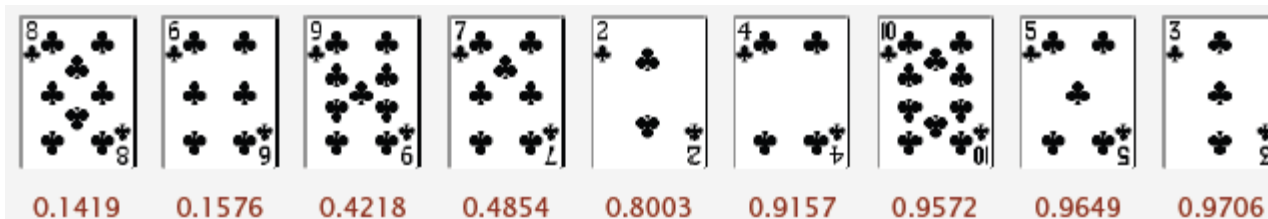
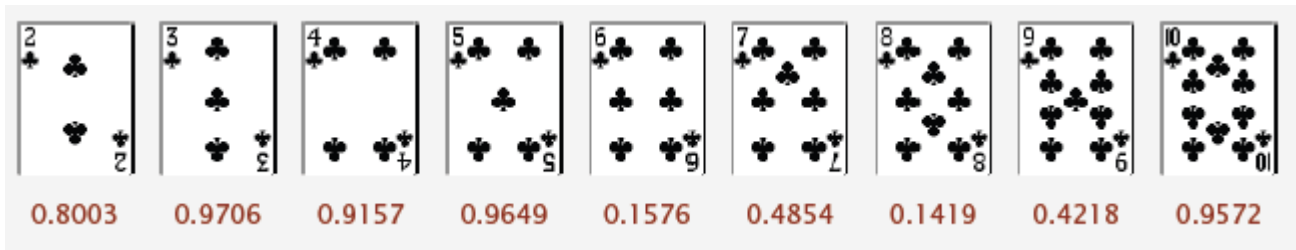
A E E L M O P R S T X

Shellsort vs. Insertion/Selection

- Simple idea leading to substantial performance gains
- Fast unless array size is huge (used for small subarrays).
- Tiny, fixed footprint for code (used in some embedded systems).
- Best sequence of increments still a mystery...

Shuffling

- Like shuffling a deck of cards – goal is to rearrange items so that they are uniformly random
- How to use sorting to do this:
 - Generate random number for each item
 - Sort based on random number



Not so random...

- Microsoft to provide randomised choice of browser for Windows 7



More Shuffling

- Knuth (author: The Art of Programming) provided another solution:
- For an array $a[n]$ of items.
 - In iteration i , pick integer r between $0-i$
 - Swap $a[i]$ and $a[r]$
- Produces uniform random array
- Efficiency class: $O(n)$ – linear.
- <http://algs4.cs.princeton.edu/lectures/21DemoKnuthShuffle.mov>

Code

```
public class StdRandom
{
    ...
    public static void shuffle(Object[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int r = StdRandom.uniform(i + 1);
            exch(a, i, r);
        }
    }
}
```

← between 0 and i

Got to be careful with shuffles

- Online poker (www.planetpoker.com) got it wrong...
- http://www.cigital.com/papers/download/developer_gambling.php

Shuffling algorithm in FAQ at www.planetpoker.com

```
for i := 1 to 52 do begin
  r := random(51) + 1;
  swap := card[r];
  card[r] := card[i];
  card[i] := swap;
end;
```

← between 1 and 51

- Bug 1. Random number r never 52 \Rightarrow 52nd card can't end up in 52nd place.
- Bug 2. Shuffle not uniform (should be between 1 and i).
- Bug 3. `random()` uses 32-bit seed \Rightarrow 2^{32} possible shuffles.
- Bug 4. Seed = milliseconds since midnight \Rightarrow 86.4 million shuffles.

Leave randomness it to the
Hardware...

